A Study on Gimbal Motion Control System Design based on Super-Twisting Control Method

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Controlling an optical sensor’s line of sight (LOS) with an inertial stabilization system carried out on a dynamic platform is a challenging engineering task. The LOS needs to track a target object accurately despite intentional maneuvers, inadvertent motions, and additional disturbances. In this study, a super-twisting sliding mode controller (STSMC) is implemented to overcome this problem. The controller is designed based on the analysis of system dynamics. The stability is then proved to be satisfactory by the Lyapunov theory. Then, the control law is validated through experimental studies. In addition, a comparison to the performance of a linear controller is derived so that the effectiveness of the proposed controller is validated.

1. Introduction

The main goal of an inertial stabilization is to hold or control the LOS of one object relative to another object or inertial space (Hilkert). It is used in a large range of applications, such as surveillance, target tracking, communications, handheld cameras, etc. The optical sensor is carried by a gimbaled structure with at least two orthogonal axes. In direct stabilization system, the sensor is considered as the payload mounted in the inner gimbal channel where its absolute motion is sensed by a gyroscope and the gimbal motions are actuated by two independent servo systems. When disturbances affect the LOS are attenuated, the system can achieve accurate tracking performance with a well-designed controller. In the case of the gimbal carried on a mobile vehicle, disturbances arise from both intentional maneuvers and inadvertent motions of the vehicle. In addition, external loads, such as wind, friction, and mass unbalance, add unpredicted perturbations into the system.

Modern control techniques based on sliding mode control are well known as robust and effective methods to be applied in these situations.
situations. However, chattering is the main drawback of the technique. In applications of inertial stabilization, many approaches based on sliding mode control have been proposed. For example, in the works of Erdoğan and Li, the use of an integral sliding mode controller combined with a disturbance/uncertainty estimator or a state observer were suggested. In Mao and Suoliang, controllers were designed based on terminal sliding mode algorithm. A combination of back-stepping control and sliding mode control was proposed in Dong and Ding.

Besides that, super-twisting algorithm not only provides finite-time convergence but also reduces chattering effects (Chalanga). Tran showed that super-twisting algorithm could be applied for inter-connected systems and performed well. Chanlanga studied an output feedback stabilization of perturbed double-integrator systems using super-twisting control. Two methodologies were proposed, one was super-twisting controller based on super-twisting observer, and the other was high-order sliding mode observer based super-twisting controller. While the first approach was proved mathematically that it was impossible to achieve second order sliding on the control sliding variable, the second approach could achieve continuous control with super-twisting algorithm. Applying these results, Reis, et al., designed a sliding mode control strategy for both stabilization and target tracking for 3-DOF inertial stabilization (Reis). Both state and output feedback cases were considered. The full state feedback values were represented in form of quaternions, while a high-order sliding mode observer was proposed to estimate velocities from the outputs. In each case, two super-twisting controllers were employed in cascade topology, providing robust and effective performance evaluated by simulation results. On the other hand, the main difficulty with super-twisting technique is to establish the proof of stability for perturbed systems. Studies of Levant, Moreno and other authors have suggested several rules for tuning controller gains such that system becomes stable. Especially, a recent study from Seeber provided a strict Lyapunov function which extends the provably stable parameter range, even for the system with unmatched disturbances.

However, above-mentioned studies related to super-twisting sliding mode control are either theoretical or simulation ones. In order to tackle effectively the problem of tracking in two-axis gimbal system, a practical approach for design and implement of the super-twisting sliding mode controller is needed. In this work, the problem is solved and presented as follows. Section 2 derives system’s mathematical model from the analysis of its kinematics and dynamics. Section 3 presents the procedure of super-twisting controller design from the system’s model. Section 4 provides stability proof encouraged from the work of Seeber and Horn. The system with proper choice of parameters is proved to be finite-time stable in the sense of Lyapunov stability theory despite of the present of disturbances. Section 5 suggests some modifications improving the robustness and effectiveness of the implemented controller. Experimental studies validating the proposed controller are discussed in Section 6. Experimental results are analyzed and compared in this section. Finally, conclusions will be drawn.

2. System Modeling

Consider a two-axis gimbal mounted on a dynamics platform illustrated in Fig. 1. The gimbal consists of an outer channel actuating pan motion and an inner channel operating tilt motion. The first step of controller design process for this system is to build up the mathematical model which characterizes the kinematics and dynamics of it. By using the Euler matrix for rotations, the angular rates of the pan and tilt channels can be respectively written as Eq. (1).

\[
\begin{align*}
\omega_x &= \begin{bmatrix}
0 & S\theta & 0
\end{bmatrix} v
+ \begin{bmatrix}
0 & 0
\end{bmatrix} \omega_n
+ \begin{bmatrix}
0 & 0
\end{bmatrix} \psi
+ \begin{bmatrix}
0 & 0
\end{bmatrix} \omega_b
+ \begin{bmatrix}
0 & 0
\end{bmatrix} \phi
+ \begin{bmatrix}
0 & 0
\end{bmatrix} \omega_c
\end{align*}
\]

Fig. 1 A two-axis gimbal mounted on a vessel
\( \omega_b = [\omega_{bx} \ \omega_{by} \ \omega_{bz}]^T \) is the angular rate of the platform, \( \dot{\psi} \) is the relative rate between the outer gimbal and the platform along Z-axis, and \( \dot{\theta} \) is the relative rate between the inner and outer gimbal along Y-axis. Next, the gimbal dynamics can be derived from the torque relationships about the inner and outer gimbal body axes. Euler's equation for rigid body dynamics is given by Eq. (2).

\[
J \ddot{\omega} + \omega \times (J \omega) = T
\]  

(2)

Where \( T \) is the applied torque, \( J \) is the matrix of inertia, and \( \omega \) is the angular velocity.

To simplify the analysis, assume that the inner gimbal rotation axes are aligned with the principal axes of inertia so that its inertia matrix is diagonal \( J = \text{diag}\{J_x, J_y, J_z\} \) and assume \( J_x = J_z \).

Applying Eqs. (1) and (2), the inner gimbal dynamics about Y-axis and the outer gimbal dynamics about Z-axis are expressed as Eqs. (3) and (4).

\[
J_x \ddot{\phi}_v + K_p \phi_v + Q_p \phi_v = T_y - T_{aw}
\]

(3)

\[
(J_x + J_m) \ddot{\phi}_p + (K_p + Q_p + Q_m) \ddot{\phi}_m + Q_p \phi_m = T_z, C \theta - T_{aw}
\]

(4)

Where \( t_y \) and \( p_z \) are index terms denoting elements corresponding to tilt motion about Y-axis and pan motion about Z-axis respectively. \( \phi \) stands for angular position of LOS, \( J \) is inertia matrix element, \( K \) and \( Q \) are viscous friction coefficient and cable restrain coefficient respectively. \( T \) is control torque and \( T_d \) indicates additional disturbance torques. Disturbances in each channel consist of mass unbalance torque \( T_{m} \), non-linear parts of friction \( T_{f} \) and cable restrain \( T_{c} \) torques, platform maneuver torques transmitted to the channel \( T_{h} \), and mutual interference between two channels (Kennedy, Ekstrand and Mokbel). \( T_{aw} \) is the total disturbance.

\[
T_{aw} = T_{aw} + T_{aw} + T_{aw} + T_{aw} + T_{aw} + T_{aw}
\]

(5)

For the sake of simplification, let us consider the case that the relative angle between the inner and outer gimbal does not change when the outer gimbal is moving. Hence, Eq. (4) becomes Eq. (6).

\[
(J_x + J_m) \ddot{\phi}_p + (K_p + Q_p + Q_m) \ddot{\phi}_m + Q_p \phi_m = T_z, C \theta - T_{aw}
\]

(6)

3. Control System Design

3.1 Tracking Control System

Typical control configurations for inertial stabilization use two control loops: inner stabilization loop and outer tracking loop (Masten). With the use of a simple P controller for stabilization, the control torques applied to actuators are generated as Eq. (7).

\[
T_y = P_y(\dot{\theta}_y - \dot{\psi}_y)
\]

(7)

Where \( P \) are the gains of inner loop controller, \( \dot{\theta}_y \) and \( \dot{\psi}_y \) are desired relative angular rate. Thus, \( \dot{\theta}_y \) and \( \dot{\psi}_y \) are the references of inner control loop, which are also the control signals from outer tracking controller. Substituting Eq. (7) into Eqs. (3) and (6), and consider Eqs. (1) and (2) yields that:

\[
J_x \ddot{\phi}_v + (K_p + Q_p) \ddot{\phi}_v + Q_p \phi_v = T_y - T_{aw} - P_y (\alpha_{\omega y} \dot{\theta}_y + \alpha_{\omega z} \dot{\psi}_y)
\]

(3)

\[
J_x \ddot{\phi}_m + (K_p + Q_p + Q_m + Q_p) \ddot{\phi}_m + Q_p \phi_m = T_z, C \theta - T_{aw} - P_y (\alpha_{\omega y} \dot{\theta}_y + \alpha_{\omega z} \dot{\psi}_y) - P_z, C \theta \frac{(\alpha_{\omega y} C \theta + \alpha_{\omega z} \dot{\phi}_m) S \theta}{C \theta} + \alpha_{\omega m}
\]

(8)

Therefore, the procedure of tracking controllers design considers Eq. (8) as the plant, which contains a closed-loop system consisted of the gimbal plant and stabilization controller.

3.2 Super-Twisting Sliding Mode Controller Design

The idea behind sliding mode control algorithm is to design a feedback control law such that control error remains on well behaved sliding manifold despite the presence of the model imprecision and of disturbances. In order to design the tracking controller for the gimbal, let us consider the general description of each channel rewritten as follow Eq. (9).

\[
\ddot{\phi}_i + a_i \dot{\phi}_i + b_i \phi_i = c_i u_i - T_{di}
\]

(9)

Where \( i = 1, 2 \) are the index of tilt channel and pan channel respectively; \( a, b, c \) and \( c \) are nominal parameters of the system model corresponding to system dynamics, \( u \) is the control signal of proposed tracking controller and \( T_{di} \) is the total disturbance.

Firstly, a sliding manifold for each channel is assigned as follow Eq. (10).

\[
\ddot{\phi}_i - k_i \phi_i - k_{i+1} \phi_i = 0
\]

(10)

Where \( k_i \) are positive definite function of time. Then, the sliding manifold is transformed to be a sliding manifold of the phase plane shown Eq. (11).

\[
\ddot{\phi}_i + a_i \dot{\phi}_i + b_i \phi_i = c_i u_i - T_{di}
\]

(11)

Finally, a sliding manifold for each channel is assigned as follow Eq. (12).

\[
\ddot{\phi}_i - k_i \phi_i - k_{i+1} \phi_i = 0
\]

(12)
Where \( e_i = \phi_i - \phi_{i0} \) is the tracking error - the difference between desired position \( \phi_{i0} \) and actual position \( \phi_i \). With \( m_i > 0 \), \( s_i \) is Hurwitz.

Take the time derivative of \( s_i \) and substitute from Eq. (9).

\[
\dot{s}_i = \dot{\phi}_i + m_i \dot{\epsilon}_i
\]

(10)

\[
= \theta_{i0} - \phi + m_i \dot{\epsilon}_i
\]

(11)

\[
= \theta_{i0} - c_i \dot{u}_i + a_i \dot{\phi}_i + b_i \phi_i + m_i \dot{\epsilon}_i + T_{si}
\]

Then, a feedback control law is selected based on Super-Twisting algorithm in order to satisfy sliding condition, \( \dot{s}_i = 0 \), taking present of disturbance into account, as follows Eq. (12).

\[
u_i = \frac{1}{\lambda_i}(u_{sem} + u_{tr})
\]

where

\[
u_{sem} = \theta_{i0} + a_i \dot{\phi}_i + b_i \phi_i + m_i \dot{\epsilon}_i
\]

\[
u_{tr} = \lambda_i \sqrt{12} \text{sign}(s_i) + z_i
\]

(13)

\[z_i = \lambda_i \text{sign}(s_i)\]

with \( \lambda_1 \) and \( \lambda_2 \) are positive constants so that the system is stable and well performance.

Substituting the control law into equation of sliding manifold surface yields that Eq. (14).

\[
\dot{s}_i = -\lambda_i \sqrt{12} \text{sign}(s_i) + x_i + T_{si}
\]

\[
x_i = -\lambda_i \text{sign}(s_i) + T_{si}
\]

(14)

\[T_{si} = T_{sem} + T_{tr} \]

denotes that in general, unknown disturbances consist of matched disturbances \( T_{sem} \) and unmatched disturbances \( T_{tr} \).

4. System Stability Analysis

Assume that the unmatched disturbances can be expressed as \( \eta |\dot{s}_i|^{1/2} \), and disturbances are bounded by non-negative constants \( \eta \leq N \) and \( |T_{sem}| \leq L \). Consider the Lyapunov candidate as Eq. (15).

\[
V = \begin{cases} 
2\sqrt[3]{\eta^2 + 3\alpha_i^2 \lambda_i^2 |s_i|} - x_i \text{sign}(s_i) \quad x_i \text{sign}(s_i) \leq \alpha_i \lambda_i \sqrt{\eta} \\
3|\dot{s}_i| \quad \text{otherwise}
\end{cases}
\]

(15)

where \( \alpha_i \) is a positive parameter. This function is continuous, positive definite, piecewise differentiable and locally Lipschitz continuous everywhere except in the origin (Seeber).

In detail, in the first case, in the subdomain of semi-positive values of \( s_i \), the function \( 2\sqrt[3]{\eta^2 + 3\alpha_i^2 \lambda_i^2 |s_i|} - x_i \) is used, and in the subdomain of negative values of \( s_i \), the function \( 2\sqrt[3]{\eta^2 - 3\alpha_i^2 \lambda_i^2 |s_i| + x_i} \) is used. Both are continuously differentiable. The similar conclusion is obtained for the other case. Therefore, \( V \) is piecewise differentiable.

In the first case, \( x_i \text{sign}(s_i) \leq \alpha_i \lambda_i \sqrt{\eta} \). Consider \( s_i \geq 0 \), and take time derivative of \( V \).

\[
\dot{V} = (\lambda_i - T_{si}) + \frac{3\alpha_i^2 \lambda_i^2 (x_i - \lambda_i \sqrt{\eta} + m \sqrt{\eta}) - 2\sqrt[3]{\eta} (\lambda_i - T_{si})}{\sqrt{x_i^2 + 3\alpha_i^2 \lambda_i^2 s_i}}
\]

(16)

\( V \) is a homogeneous function of degree zero with respect to \( \sqrt{x_i^2 + 3\alpha_i^2 \lambda_i^2 s_i} \) and \( x_i \). Then, the function’s behavior for values of \( x_i \) satisfying \( x_i^2 + 3\alpha_i^2 \lambda_i^2 s_i = 1 \) should be hold for any greater range of \( x_i \). One consequently, defines the function.

\[
g(x_i) = \left. \frac{\alpha_i \lambda_i}{1 - x_i^2} \text{sign}(s_i) \right|_{x_i^2 + 3\alpha_i^2 \lambda_i^2 s_i} \]

(17)

The restrictions \( s_i \geq 0 \) and \( x_i \leq \alpha \lambda_i \sqrt{\eta} \) are equivalent to the argument \( x_i \) of this function lying in the interval \([-1, 1/2]\]. The second derivative of Eq. (17) is:

\[
\frac{d^2 g}{d x_i^2} = \left. \frac{\alpha_i \lambda_i}{(1 - x_i^2)^{3/2}} \right|_{x_i^2 + 3\alpha_i^2 \lambda_i^2 s_i} (1 + 2 x_i^2) (\lambda_i - \eta)
\]

(18)

It can be seen that with \( \lambda_i \geq N, d^2 g/d x_i^2 \geq 0 \), then \( g(x_i) \) is convex. Therefore:

\[
g(x_i) \leq \max \left( g(-1), g \left( \frac{1}{2} \right) \right)
\]

(19)

where

\[
g(-1) = 3\lambda_i - T_{si} - \alpha_i \lambda_i \leq 3\lambda_i + L - \alpha_i \lambda_i
\]

\[
g \left( \frac{1}{2} \right) = \frac{3}{2} \alpha_i \lambda_i (\lambda_i - \alpha_i \lambda_i + \eta)
\]

(20)

Thus, by homogeneity one concludes that:

\[
\dot{V} \leq \max \left( 3\lambda_i + L - \alpha_i \lambda_i, \frac{3}{2} \alpha_i \lambda_i (\alpha_i \lambda_i - \eta) \right)
\]

(21)

The system stability is preserved if \( \dot{V} < 0 \), then controller gains must satisfy

\[
0 < \alpha_i < 1 - \frac{\eta}{\lambda_i}
\]

(22)

and

\[
\lambda_i > N + \sqrt{\lambda_i^2 + L}
\]

(23)
If $s_i < 0$, the same conclusion is obtained by a similar procedure. Furthermore, in the case of $x_i \text{sign}(s_i) > \alpha_i \hat{\lambda}_i \sqrt{\int_{0}^{t} |s| \, dt}$ time derivative of $V$ yields that:

$$
\dot{V} = 3 \text{sign}(x_i) \left( -\lambda_2 \text{sign}(s_i) + \dot{s}_\text{on} \right) \\
\dot{V} = 3 \text{sign}(x_i) \left( -\lambda_2 \text{sign}(s_i) + \dot{s}_\text{on} \right) < 3(L - \lambda_2)
$$

(24)

From Eqs. (23) and (25), the system is finite time stable if the conditions $\lambda_2 > L$ and $\lambda_1 > N + N \lambda_2 + L$ are satisfied.

5. Implementation

Chattering is well known as one of the main disadvantages of sliding mode controller. It has been shown that this effect is mainly caused by unmodelled cascade dynamics which increase the system’s relative degree and perturb the ideal sliding mode existing in the system (Fridman\textsuperscript{19}).

By including an integrator, the Super-Twisting algorithm is able to attenuate chattering in relative degree one system. In order to reduce the influence of higher relative degree, the proposed control law is calculated from a continuous signum-like function instead of the sign function.

$$
(u_{st})_{ij} = \lambda_{11} \left| s_i \right|^2 \text{sgn}(s_i, \delta_i) + z_i \\
\dot{z}_i = \lambda_2 \text{sgn}(s_i, \delta_i)
$$

(26)

Where $\text{sgn}(s_i, \delta_i)$ is defined as $\text{sgn}(s_i, \delta_i) = \frac{s_i}{|s_i| + \delta_i}$, $\delta$ is a positive constant.

6. Experiment

Experimental studies are performed on a two-axis gimbal used in ocean surveillance applications. Nominal parameters of the system defined in Eq. (9) are experimentally obtained using system identification method. Their values are presented as follows.

$$\begin{align*}
a &= 21.97, & b &= 0.2757, & c &= 22.25 \\
\alpha_2 &= 30.4, & b_2 &= 0.02994, & c_2 &= 28.44
\end{align*}$$

System configuration is shown in Fig. 3. The payload is mounted at the center of tilt gimbal. Its absolute angular position and rate are measured by an Attitude Heading Reference System (AHRS) attached directly.

Two channels of the gimbal are actuated independently by two servo system through driving belts. There is already a P controller for speed control in each servo as mentioned in Section 3. All actuators and sensor communicate through RS 232 interface. The controller is implemented in Matlab/Simulink. Sampling time is chosen as 0.05s. The hardware system specifications are listed in Table 1, and the controller gains are represented in Table 2.
At first, two channels are experimented to track target trajectories in forms of step signal value. The response of tilt gimbal and its control signal are shown in Figs. 4(a) and 4(b). Experimental results collected from pan motion are shown in Figs. 4(c) and 4(d). Note that the control signals of the tracking controller are the reference values for the inner stabilizer controllers, so their units are deg/s. One can see that the proposed method achieves desired tracking position in a short time with almost no overshoot. The zoomed figures show in detail the transient responses and steady states of each channel. Evidently, the system is robust and well performed.

Then, a comparison of system performance controlled by STSMC and PID controller is presented to evaluate the efficiency of the proposed control law more clearly. The PID controller has the general form as follows Eq. (27).

\[
G_i(s) = P + T_i \frac{1}{s} + D_i - \frac{N_i}{1 + \frac{1}{s}}
\]  

where tuned controller gains are also represented in Table 2.

Both channels of the gimbal are controlled at the same time so that the LOS tracked a target object following a counterclockwise-circular trajectory in a vertical plane. Tracking performances are shown in Fig. 5. Furthermore, the root-mean-square errors (RMS) of the experiment data are calculated and shown in Table 3.

In Fig. 5(a), the circular trajectory in the vertical plane and corresponding system outputs are presented. At the first glance on the results, the PID control seems to achieve better tracking performance. However, it is worth to notice that the results are shown geometrically only in Fig. 5(a), which may lead to the difficulty in evaluating the system responses with respect to time. The zoomed figures, where the reference point and corresponding system outputs at a specific time are highlighted, show that the tracking error by STSMC is smaller than the error generated by PID controller. It is easy to see that the system response controlled by STSMC is very close to the reference, while the one by the PID controller is always far behind. The tracking errors in Fig. 5(b) also prove that the STSMC archives significantly better performance.

In detail, Fig. 5(c) shows the angle response of each channel controlled by PID controller is always behind and the one of STSMC is almost match to the reference. In addition, Fig. 5(d) shows the control signals from STSMC and PID controller. The element \( u_{st} \) in the STSMC’s control signal compensates unmodelled dynamics and disturbances perturbed the system, which causes the bounce as seen in this Fig. 5.

For the RMS tracking errors shown in Table 3, the proposed STSMC provides much smaller tracking error in both channels. However, the radial residual of PID controller is accidentally smaller, due to periodic characteristic of circular trajectory.
7. Conclusions

In this paper, a super-twisting sliding mode controller has been designed for the gimbal motion control. A system model has been obtained from the analysis of system kinematics and dynamics, and consideration of the presence of stabilization controller. By experimental studies, the effectiveness of the proposed controller has been shown and validated. Comparing with tradition PID controller, the designed super-twisting controller has achieved much better performance. In detail, for the tilt and pan channels, the RMS error of the proposed control method is respectively equal to 5.7 and 24% of those from PID control. The system performance robustness with respect to limited disturbances is proved by Lyapunov theory. Consider complex kinematics coupling and outer disturbance influencing the system, future studies are necessary to enhance system performance and robustness.

Table 3 RMS values

<table>
<thead>
<tr>
<th></th>
<th>Tilt</th>
<th>Pan</th>
<th>Radial</th>
</tr>
</thead>
<tbody>
<tr>
<td>STSMC</td>
<td>0.0017</td>
<td>0.0071</td>
<td>0.0043</td>
</tr>
<tr>
<td>PID</td>
<td>0.0298</td>
<td>0.0295</td>
<td>0.0025</td>
</tr>
</tbody>
</table>

Fig. 5 Tracking performance for a circular reference trajectory

(b) Tracking error of the tilt (above) and the pan (Bellow)

(c) Angle of the tilt (Top) and the pan (Bottom)

(d) Control signal for the tilt (Top) and the pan (Bottom)
REFERENCES


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